Least-squares refinement of centrosymmetric trial structures in non-centrosymmetric space groups. A warning.

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It is impossible to distinguish by means of a least-squares analysis between a centrosymmetric structure and a corresponding non-centrosymmetric one by simple expansion of the set of parameters over the questionable inversion centre.

In many cases the space group of a crystal is not uniquely defined by the systematically absent reflexions. In solving the structure of such a crystal from X-ray diffraction data it is often convenient to start from the simplifying assumption that the structure is centrosymmetric, but once a reasonable trial structure has been found on this basis the question arises as to whether the actual structure is centrosymmetric or not. If one wishes to answer this question by least-squares analysis of the diffraction data, one might decide to simply expand the trial set of parameters over the questionable inversion centre and carry out the leastsquares refinement in the corresponding non-centrosymmetric space group. This procedure is easily shown to be invalid.

Since the structure factor derivatives with respect to pairs of centrosymmetrically related positional parameters are equal in magnitude and have opposite sign, while derivatives with respect to corresponding pairs of temperature factor parameters are equal, the normal equations are identical in pairs and the resulting normal equations matrix becomes singular. In such a case, full-matrix refinement would lead to catastrophic results, while diagonal or block-diagonal refinement is clearly equivalent to refinement in the originally assumed centrosymmetric space group. Small, random shifts may be applied to the centrosymmetric set of parameters so as to make it only approximately centrosymmetric, but then the occurrence of an ill-conditioned set of normal equations has to be reckoned with. Diamond (1958) has shown how an eigenvalueeigenvector technique may be applied to obtain the maximum amount of information in similar cases.

Reference

DIAMOND, R. (1958). Acta Cryst. 11, 129.

Acta Cryst. (1970). A 26, 163

Magnetic symmetry and transport properties of crystals. By P.V. PANTULU and E. SUDARSHAN, Scientific Adviser's Secretariat, 177-A. South Block, New Delhi-11, India

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The generalized Onsager relations applicable to transport property tensors for magnetic and non-magnetic crystals in the presence or absence of an external magnetic field were given by Kleiner. In this paper it is shown that the symmetry-restricted forms of the thermogalvanomagnetic property tensors conforming to Kleiner's prescription can be obtained from the forms of the polar and axial tensors appropriate to the 32 classical point groups, making use of the rules given by Birss for the equilibrium magnetic property tensors.

We consider the effect of magnetic symmetry of a crystal on its thermogalvanomagnetic (TGM) properties. The electric current density J_i , the heat current density q_i , the electric field E_i and the negative temperature gradient G_i in a crystal are related as shown in the phenomenological equations (1). The usual summation convention has been adopted throughout this paper.

$$E_{i} = \varrho_{ij}J_{j} + \alpha_{ij}G_{j},$$

$$q_{i} - J_{i} \frac{\xi}{e} = -\beta_{ij}J_{j} + \kappa_{ij}G_{j}.$$
(1)

 ξ is the chemical potential of the electrons, *e* is the electronic charge, ρ_{ij} is the electrical resistivity, κ_{ij} the thermal conductivity, α_{ij} the thermoelectric power and β_{ij} the property inverse to α_{ij} .

In the presence of an external magnetic field, the tensors ρ_{ij} , α_{ij} , β_{ij} and κ_{ij} can be expanded as power series in the field components H_i as in (2).

$$\varrho_{ij}(\mathbf{H}) = \varrho_{ij} + \varrho_{ijk}H_k + \varrho_{ijkl}H_kH_l + \dots$$
(2)

The tensors of various ranks on the right-hand side of (2) define the TGM tensors. H_i is an axial vector whereas J_i , q_i , E_i and G_i are polar vectors. Consequently the TGM tensors of even rank are polar while those of an odd rank are axial.

According to Kleiner (1966) the space-time symmetry of a crystal, in which these properties are observed, imposes the relations (3) and (4) between the corresponding tensor components. These relations which take account of the space-time symmetry of the crystal are the appropriate generalizations of the classical Onsager relations.

A pure rotation in space is represented by a 3×3 orthogonal matrix $||R_{ij}||$. The space-time operation of a pure rotation followed by time-invariance is denoted by R, and that followed by time-reversal by R. A rotation-inversion followed by time-invariance is denoted by R while that followed by time-reversal is denoted by R.

For R or R:
$$\varrho_{ijkl} \ldots = R_{im}R_{jn}R_{kp}R_{lq} \ldots \varrho_{mnpq}$$
. (3a)